a review of Noncommutative geometry, quantum fields and motives by Connes, Alain; Marcolli, Matilde

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The book, consisting of 4 chapters, is concerned with the deep interplay between noncommutative geometry and number theory, or more specifically, between noncommutative spaces and motives. The authors' humongous aspiration is to show the relevance of noncommutative geometry to the challenging problem of the construction of a quantum gravity on the one hand and that of the Riemann hypothesis on the other.

The two main topics of Chapter 1 are renormalization and the standard model of high energy physics. The first part of Chapter 1, which is concerned with renormalization, is devoted to giving a new account of the perturbative quantum field theory, which gives a clear mathematical interpretation of the renormalization procedure used by physicists to extract finite values from the divergent expressions as a result of the evaluations of the integrals associated to so-called Feynman diagrams. This part is based upon [*A. Connes* and *D. Kreimer*, Commun. Math. Phys. 210, No. 1, 249–273 (2000; Zbl 1032.81026) and Commun. Math. Phys. 216, No. 1, 215–241 (2001; Zbl 1042.81059)].

The main new result of the first part of Chapter 1 is the explicit identification of the Riemann-Hilbert correspondence secretly present in perturbative renormalization. The principal objective in the second part of Chapter 1 is to show that the intricate Lagrangian of the standard model minimally coupled to gravity can be completely derived from very simple mathematical data by using the formalism of noncommutative geometry. The authors' approach can be thought of as a unification based upon the symplectic unitary group in Hilbert space, rather than upon finite-dimensional Lie groups. The model provides specific values of some of the parameters of the standard model at unification scale by running them down to ordinary scales through the renormalization group via the Wilsonian approach. In particular, the arbitrary parameters of the standard model as well as those of gravity acquire a clear geometric meaning in terms of moduli spaces of Dirac operators on the noncommutative geometry and of the asymptotic expansion of the corresponding spectral action functional. In the last section of Chapter 1, the authors return to the initial discussion of perturbative renormalization and dimension regularization, constructing natural noncommutative spaces X_z for a complex number z as dimension in the sense of the dimension spectrum of spectral triples, which gives a concrete geometric meaning to the dimensional regularization procedure. It is shown that the algebraic rules of t'Hooft-Veltman and Breitenlohner-Maison on how to handle chiral anomalies by using the dimensional regularization procedure are obtained, as far as one-loop fermionic graphs are concerned, as a result of the inner fluctuations of the metric in the product by the spaces X_z.

Following the first author [Sel. Math., New Ser. 5, No. 1, 29–106 (1999; Zbl 0945.11015)], Chapter 2 describes a spectral realization of the zeros of the Riemann zeta function and an interpretation of Weil's explicit formulae of number theory as a trace formula. Following *A. Connes, C. Consani* and *M. Marcolli* [Adv. Math. 214, No. 2, 761–831 (2007; Zbl 1125.14001)], the chapter also deals with an application of the same techniques to the Archimedean local factors of L-functions of varieties over number fields. The authors conclude the chapter with an analogy between the real mixed Hodge structures involved in the definition of the Archimedean local L-factors and their motivic Galois group and the category of flat equisingular connections used in the previous chapter in the context of perturbative renormalization, and their differential Galois group $\mathbb{U}^*$.

Following [*J.-B. Bost* and *A. Connes*, Sel. Math., New Ser. 1, No. 3, 411–457 (1995; Zbl 0842.46040); *A. Connes* and *M. Marcolli*, in: Frontiers in number theory, physics, and geometry I. On random matrices, zeta functions, and dynamical systems. Papers from the meeting, Les Houches, France, March 9–21, 2003. Berlin: Springer. 269–347 (2006; Zbl 1126.58006); J. Geom. Phys. 56, No. 1, 2–23 (2006; Zbl 1139.58003)], [*A. Connes, M. Marcolli* and *N. Ramachandran*, Sel. Math., New Ser. 11, No. 3–4, 325–347 (2005; Zbl 1106.58005) and in: Operator algebras. The Abel symposium 2004. Proceedings of the first Abel symposium, Oslo, Norway, September 3–5, 2004. Berlin: Springer. Abel Symposia 1, 15–59 (2006;

Zbl 1123.58004)], the authors consider more general types of noncommutative adelic quotients and their relation to Galois theory in Chapter 3. All the cases discussed in this chapter are quantum statistical mechanical systems with nontrivial phase transition phenomena and with thermodynamical equilibrium states which recover the points of a classical algebro-geometric moduli space at sufficiently low temperature. Towards the end of the chapter, the authors also discuss some further generalizations to the case of Shimura varieties on the lines of *E. Ha* and *F. Paugam* [IMRP, Int. Math. Res. Pap. 2005, No. 5, 237–286 (2005; Zbl 1173.82305)] and to function fields on the lines of *B. Jacob*, [J. Noncommut. Geom. 1, No. 2, 141–211 (2007; Zbl 1176.46061)] and *C. Consani* and *M. Marcolli* [J. Number Theory 123, No. 2, 487–528 (2007; Zbl 1160.11043)], and the relation of the GL_2-system to the modular Hecke algebras of *A. Connes* and *H. Moscovici*, on the lines of [Mosc. Math. J. 4, No. 1, 67–109 (2004; Zbl 1122.11023); Mosc. Math. J. 4, No. 1, 111–130 (2004; Zbl 1122.11024); in: Noncommutative geometry and number theory. Where arithmetic meets geometry and physics. Based on two workshops, Bonn, Germany, August 2003 and June 2004. Wiesbaden: Vieweg. Aspects of Mathematics E 37, 79–107 (2006; Zbl 1109.11027)].

Based on *A. Connes, C. Consani* and *M. Marcolli* [Adv. Math. 214, No. 2, 761–831 (2007; Zbl 1125.14001) and in: Algebra, arithmetic, and geometry. In honor of Yu. I. Manin on the occasion of his 70th birthday. Vol. I. Boston, MA: Birkhäuser. Progress in Mathematics 269, 339–405 (2009; Zbl 1208.11108)], Chapter 4 clarifies the conceptual meaning of the spectral realization in terms of noncommutative geometry and cyclic cohomology.

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MSC:

58B34 Noncommutative geometry (à la Connes)
58-02 Research exposition (monographs, survey articles) pertaining to global analysis
11-02 Research exposition (monographs, survey articles) pertaining to number theory
81-02 Research exposition (monographs, survey articles) pertaining to quantum theory
11G35 Varieties over global fields

Keywords:

noncommutative geometry; renormalization; Feynman diagram; motive; quantum gravity; number theory; Riemann hypothesis; Bost-Connes system; adèle class space; category; zeta function; Archimedean place; Galois group; Hamiltonian; Lagrangian; Hodge structure; KMS state; Hopf algebra; KK-theory; Kasparov module; L-function; Majorana mass term; modular field; Morita equivalence; moduli space; noncommutative space; KO-dimension; perturbation theory; Haar measure; quarks; scaling; singularities; Tannaka formalism; trace formula; Weil explicit formula; Yang-Mills theory; unification scale; triangulated category; standard model; spectral triple; spectral realization; principal divisors; Pntecorvo-Maki-Nakagawa-Sakata matrix; Pontrjagin dual; partition function; Lefschetz trace formula; Higgs field; Hecke algebra; GNS representation; Galois action; Fourier transform; dimension spectrum; coupling constants; color index; bosons; fermions; arithmetic subalgebra; Birkhoff factorization; Artin motives; cyclic cohomology; Riemannian spin manifold; Riemann-Hilbert correspondence; Hochschild cohomology; $\mathbb{K}$-lattice; idèle class group; Artin motives; motivic cohomology; Weil group